Exercise 1: The Solow Growth Model

Consider the following economy (for simplicity, time indexes are omitted): The production function is

$$Y = K^{\alpha} (AL)^{1-\alpha}$$

where K, L, and A stand for aggregate capital, labor and knowledge, respectively. Labor and knowledge grow at constant exogenous rates n and g. The savings rate, s, and the depreciation rate δ are both constant.

1. Derive the steady-state level of capital per effective labor k and output per effective labor y as a function of the various exogenous parameters.
2. Derive the golden-rule level of capital per effective labor and output per effective labor as a function of the various exogenous parameters.
3. Derive the savings rate required to attain the golden-rule level of capital per effective labor.
4. Now consider two countries, A and B. Both have the same production function as given above and all exogenous parameters are the same for both countries except population growth which is higher in country A. Illustrate the steady states of both economies graphically. Based on the information that you have, can you determine whether both economies will have the same steady-state stock of capital per effective labor and output per effective labor or whether one will attain a higher steady state than the other?
5. Assume that there is a one-off migration of a large number of workers from country A to country B. Discuss the impact of this event on capital per effective labor, output per effective labor and output per worker, and their growth rates, both in the short term and in the long term, in both countries. Illustrate the impact of this event in the diagrams that you drew.