Endogenous Growth Theory

Motivation

The Solow and Ramsey models offer valuable insights but have important limitations:

- Differences in capital accummulation cannot satisfactorily account for the prevailing large differences in output per worker;
- In the steady state, the only source of growth is *technological progress*, which is assumed to be exogenous and whose determinants are not explored.

Therefore, the new growth theory developed models of *endogenous* growth which explicitly model the *production of knowledge*, or *R&D*.

Key contributions: Paul Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992, 1998).

Assumptions

Two sectors: a goods-producing sector and an R&D sector (production of knowledge). The two sectors have to share the available stocks of $K(t)$ and $L(t)$. Both sectors use the full stock of knowledge, $A(t)$.

Production function of the *goods-producing sector* is a CRTS Cobb-Douglas, using fraction $1 - a_L$ of labor and $1 - a_K$ of capital (a_L and a_K are exogenous and constant):

$$Y(t) = \left[(1 - a_K) K(t)\right]^{\alpha} \left[A(t)(1 - a_L) L(t)\right]^{1-\alpha}$$

where $0 < \alpha < 1$. Except for assuming that the production process only uses a part of the available capital and labor, this is the same as the production function used so far.

Production function of the *R&D sector* is a generalized Cobb-Douglas PF:

$$\dot{A}(t) = B\left[a_K K(t)\right]^{\beta} \left[a_L L(t)\right]^{\gamma} A(t)^{\theta}$$

where $\beta \geq 0$, $\gamma \geq 0$ and $B > 0$ is a shift parameter.

Note that R&D production is formulated in such a way that constant, diminishing or increasing returns to scale to capital and labor are both possible:

doubling of inputs may results in more frequent duplicity of discoveries (diminishing returns), or it may result in more discoveries because of fixed set-up costs, interactions between scientists, etc.

θ reflects the relationship between the existing stock of knowledge and the R&D process. No a-priori restriction is placed on its sign: $\theta > 0$ implies that the existing stock of knowledge encourages further R&D, $\theta < 0$ stands for the opposite relationship, i.e. the easiest discoveries are made first and the production of knowledge is inversely related to its stock.

The savings rate is constant exogenous and there is no depreciation:

$$\dot{K}(t) = sY(t).$$

Population growth is exogenous:

$$\dot{L}(t) = nL(t)$$

where $n \geq 0$.

That this model reduces to the standard Solow model if $a_L = a_K = 0$, $\beta = \gamma = 0$ and $\theta = 1$. Then $\dot{A}(t) = BA(t)$ so that B then becomes the rate of growth of knowledge.

The general model is analytically more challenging than the Solow model because it features two endogenous stock variables: $K(t)$ and $A(t)$. We start by considering a simplified case

Model without Capital

Accumulation of Knowledge

$$Y(t) = A(t)(1 - a_L)L(t)$$
$$\dot{A}(t) = B[a_L L(t)]^\gamma A(t)^\theta.$$

Output per worker is proportional to A, $Y(t)/L(t) = A(t)(1 - a_L)$, and the growth rate of output per worker is equal to the growth rate of $A(t)$. We therefore focus on the dynamics of A:

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = B[a_L L(t)]^\gamma A(t)^{\theta-1}.$$

Taking logs and differentiating wrt t, we get the *growth rate of the growth rate* of A:

$$\begin{aligned}
\ln g_A(t) &= \ln B + \gamma \ln a_L + \gamma \ln L(t) + (\theta - 1)\ln A(t) \\
\frac{\dot{g}_A(t)}{g_A(t)} &= \gamma n + (\theta - 1) g_A(t)
\end{aligned}$$

or

$$\dot{g}_A(t) = \gamma n g_A(t) + (\theta - 1)[g_A(t)]^2.$$

The evolution of g_A depends on the value of θ, specifically, on whether θ is smaller than, greater than or equal to 1.

Case 1: $\theta < 1$

The above is a quadratic equation in g_A. We are looking for a steady state where g_A is constant, or $\dot{g}_A = 0$. That will be the case either if $g_A = 0$ or if

$$\begin{aligned}\gamma n + (\theta - 1) g_A(t) &= 0 \\ g_A^*(t) &= \frac{\gamma}{1-\theta} n\end{aligned}$$

which is positive. Because $\theta < 1$, the second term in the quadratic expression for $\dot{g}_A$ is negative, graphically $\dot{g}_A$ is a humpshaped curve starting from 0, intersecting the horizontal axis at g_A^*, $\dot{g}_A$ is thus positive for $0 < g_A < g_A^*$ and negative for $g_A > g_A^*$. Therefore, regardless of the initial conditions, the economy converges to g_A^*. Once the economy reaches g_A^*, it is on a balanced-growth path.

Figure 3.1.

In contrast to Solow, Ramsey and Diamond models, this model displays *endogenous* growth: the steady-state growth of output per worked, g_A^*, is determined *within* the model.

Note the following properties:

- The steady-state growth rate of output per worker is proportional to population growth: technological progress requires sustained population growth and countries with faster population growth grow faster (weakness: this property does not seem to be confirmed by actual data);

- The steady-state growth rate does not depend on the fraction of labor employed in the production of knowledge, a_L. A change in a_L has a temporary level effect (because $g_A(t) = B[a_L L(t)]^\gamma A(t)^{\theta-1}$ and $\dot{A}(t) = B[a_L L(t)]^\gamma A(t)^\theta$) but does not affect the steady-state rate of growth.

Case 2: $\theta > 1$

Production of knowledge rises more than proportionally with the existing stock. Recall:

$$\dot{g}_A(t) = \gamma n g_A(t) + (\theta - 1)[g_A(t)]^2.$$

This implies that, except for $g_A = 0$, $\dot{g}_A$ is always positive (note: $\frac{\gamma}{1-\theta}n < 0$). Growth accelerates over time and the economy never converges to a balanced growth path.

Figure 3.2.

Production of knowledge is self-reinforcing: each new discovery encourages further technological progress. Any increase in a_L increases g_A permanently ($g_A(t) = B[a_L L(t)]^\gamma A(t)^{\theta-1}$) and further accelerates $\dot{g}_A$.

Case 3: $\theta = 1$

Production of knowledge is proportional to its stock. Therefore,

$$\begin{aligned} g_A(t) &= B[a_L L(t)]^\gamma \\ \dot{g}_A(t) &= \gamma n g_A(t). \end{aligned}$$

Figure 3.3.

For $n > 0$, growth again accelerates over time (albeit at a constant rate). Again, the economy never reaches a balanced-growth path.

If $n = 0$ (or if $\gamma = 0$), the economy is always on a balanced growth path: $\dot{g}_A = 0$ so that g_A is always constant. $g_A = B[a_L L(t)]^\gamma$ so that the steady-state growth rate depends on a_L.

Note that if $\theta > 1$ or if $\theta = 1$ and $n > 0$, growth is continuously increasing so that output reaches infinity in finite amount of time.

The model's implications crucially depend on whether the model displays constant, increasing or decreasing returns in the *produced* factors of production: in this case knowledge only. $\theta < 1$ implies DRTS, $\theta > 1$ corresponds to IRTS, and $\theta = 1$ is CRTS.

The General Case

$$Y(t) = [(1-a_K)K(t)]^\alpha [A(t)(1-a_L)L(t)]^{1-\alpha}$$

$$\dot{A}(t) = B[a_K K(t)]^\beta [a_L L(t)]^\gamma A(t)^\theta$$

$$\dot{K}(t) = sY(t).$$

$$\dot{L}(t) = nL(t)$$

Accumulation of Knowledge and Capital

There are two endogenous variables: $A(t)$ and $K(t)$. Their dynamics determine the properties of the model.

Substituting for Y in the $\dot{K}$ expression and dividing by K

$$\dot{K}(t) = s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha} K(t)^{\alpha} A(t)^{1-\alpha} L(t)^{1-\alpha}$$

$$\begin{aligned} g_K(t) &= s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha} K(t)^{\alpha-1} A(t)^{1-\alpha} L(t)^{1-\alpha} \\ &= s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha} \left[\frac{A(t)L(t)}{K(t)}\right]^{1-\alpha} \\ &= c_K \left[\frac{A(t)L(t)}{K(t)}\right]^{1-\alpha} \end{aligned}$$

where $c_K = s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha}$. Taking logs and differentiating wrt t:

$$\begin{aligned} \ln g_K(t) &= \ln c_K + (1-\alpha)[\ln A(t) + \ln L(t) - \ln K(t)] \\ \frac{\dot{g}_K(t)}{g_K(t)} &= (1-\alpha)[g_A(t) + n - g_K(t)]. \end{aligned}$$

$g_K(t)$ is always positive because $\dot{K}(t) = sY(t) > 0$ (no depreciation). Therefore, $g_K(t)$ is increasing ($\frac{\dot{g}_K(t)}{g_K(t)} > 0$) if $g_A(t) + n - g_K(t) > 0$, falling if $g_A(t) + n - g_K(t) < 0$, and constant if $g_A(t) + n - g_K(t) = 0$. In g_A/g_K space, $\dot{g}_K = 0$ is given by a straight line ($g_K(t) = g_A(t) + n$) with slope 1 which intersects the vertical axis at $g_K = n$.

Figure 3.4.

Dividing $\dot{A}(t)$ by $A(t)$:

$$\dot{A}(t) = B[a_K K(t)]^{\beta} [a_L L(t)]^{\gamma} A(t)^{\theta}$$

$$\begin{aligned} g_A(t) &= B a_K^{\beta} a_L^{\gamma} K(t)^{\beta} L(t)^{\gamma} A(t)^{\theta-1} \\ &= c_A K(t)^{\beta} L(t)^{\gamma} A(t)^{\theta-1} \end{aligned}$$

where $c_A = B a_K^{\beta} a_L^{\gamma}$. Taking logs and differentiating wrt t:

$$\begin{aligned} \ln g_A(t) &= \ln c_A + \beta \ln K(t) + \gamma \ln L(t) + (\theta-1)\ln A(t) \\ \frac{\dot{g}_A(t)}{g_A(t)} &= \beta g_K(t) + \gamma n + (\theta-1) g_A(t). \end{aligned}$$

$g_A(t)$ is increasing if $\beta g_K(t) + \gamma n + (\theta - 1) g_A(t) > 0$, falling if $\beta g_K(t) + \gamma n + (\theta - 1) g_A(t) < 0$, and constant if $\beta g_K(t) + \gamma n + (\theta - 1) g_A(t) = 0$. In g_A/g_K space, $\dot{g}_A = 0$ is given by a straight line ($g_K(t) = -\frac{\gamma}{\beta} n + \frac{(1-\theta)}{\beta} g_A(t)$) with slope $\frac{(1-\theta)}{\beta}$ which intersects the vertical axis at $g_K = -\frac{\gamma}{\beta} n$.

Figure 3.5.

As before, the model's implications depend on the returns to scale in the *produced* factors of production (capital and knowledge) in the production of knowledge.The production function has CRTS; the production of knowledge displays IRTS if $\beta + \theta > 1$, DRTS if $\beta + \theta < 1$, and CRTS if $\beta + \theta = 1$.

Case 1: $\beta + \theta < 1$

$\beta + \theta < 1$ implies $1 - \theta > \beta$ or $\frac{1-\theta}{\beta} > 1$ so that the $\dot{g}_A = 0$ line is steeper than the $\dot{g}_K = 0$ line.The equilibrium is given by the intersection of the two lines:

$$\begin{aligned} g_A^* + n - g_K^* &= 0 \\ \beta g_K^* + \gamma n + (\theta - 1) g_A^* &= 0. \end{aligned}$$

Figure 3.6.

Solving for g_K^* from the first equation and substituting to the second

$$\begin{aligned} \beta (g_A^* + n) + \gamma n + (\theta - 1) g_A^* &= 0 \\ (\beta + \gamma) n + (\beta + \theta - 1) g_A^* &= 0 \\ g_A^* &= \frac{(\beta + \gamma)}{1 - (\beta + \theta)} n \\ g_K^* &= g_A^* + n. \end{aligned}$$

The economy converges to a balanced-growth path, regardless of the initial conditions. In the steady state, both K and A grow at constant rates. Aggregate output grows at rate

$$\alpha g_K^* + (1-\alpha) g_A^* + (1-\alpha) n = \alpha (g_A^* + n) + (1-\alpha) g_A^* + (1-\alpha) n = g_A^* + n = g_K^*$$

Output per worker grows at rate g_A^*.

The model again displays endogenous growth; the steady-state growth rate of output per worker is proportional to population growth. a_L, a_K or s do not affect the steady-state rate of growth.

Case 2: $\beta + \theta = 1$ and $n = 0$

When $\beta + \theta = 1$, the slope of the $\dot{g}_A = 0$ line is 1, the same as the slope of the $\dot{g}_K = 0$ line.

$$\begin{aligned}
(1-\theta)\, g_K(t) + \gamma n + (\theta - 1)\, g_A(t) &= 0 \\
g_K(t) &= \frac{\gamma}{\theta - 1} n + g_A(t)
\end{aligned}$$

With $n = 0$ both lines coincide and $g_K = g_A$.

Figure 3.7.

Irrespective of initial conditions, the economy always converges to the steady state. Once the steady state, $g_K = g_A$ and the economy grows at a contant rate.

Secial case: simplified Paul Romer (1990) model:

$$\dot{A}(t) = B a_L L A(t)$$

$$Y(t) = K(t)^{\alpha} \left[(1 - a_L) L A(t) \right]^{1-\alpha}$$

$$\dot{K}(t) = sY(t)$$

Since $n = 0$, $\dot{g}_A = B a_L L$ is constant. This model is effectively a Solow-type of model, although the steady-state growth rate is endogenous.

Other cases: $\beta + \theta > 1$ or $\beta + \theta = 1$ and $n > 0$

There is no steady state because the $\dot{g}_A = 0$ and $\dot{g}_K = 0$ do not intersect: either the $\dot{g}_A = 0$ line is flatter ($\frac{1-\theta}{\beta} < 1$) than the $\dot{g}_K = 0$ or both lines are parallel but do not coincide.

Nature of Knowledge and R&D: A Closer Look

Knowledge is *non-rival*: it can be used by others without diminishing the amount of knowledge available to the original user or inventor.

Because it is *non-rival*, knowledge cannot be efficiently produced under competitive markets: the marginal cost of replicating past inventions is close to zero. Patents and protection of intellectual property rights make knowledge *excludable*, i.e. others may be prevented from using it. Excludability creates private incentives for R&D.

If knowledgeis *non-excludable*, private incentives for its production are essentially non-existent. Because of the externalities inherent in the production of knowledge, there may be strong arguments for public funding of R&D.

The ability to enjoy private returns from innovative activity is crucial for determining whether talented individuals choose to engage in entrepreneurial activity and innovation or in rent seeking (Murphy, Shleifer and Vishny, 1991, QJE).

Endogenous Growth and Learning by Doing

Knowledge is modeled as by-product of the production process rather than an outcome of a separate production process (Romer, 1990). Hence, all inputs are engaged in the goods sector:

$$Y(t) = K(t)^{\alpha} [L(t) A(t)]^{1-\alpha}$$

Savings rate and population growth are constant and exogenous:

$$\dot{K}(t) = sY(t).$$

$$\dot{L}(t) = nL(t)$$

and knowledge is created as a side product of accumulation of physical capital (learning by doing):

$$A(t) = BK(t)^{\phi}$$

where $B > 0$ and $\phi > 0$.
We can substitute for $A(t)$ in the production function and then substitute for $Y(t)$ in the $\dot{K}(t)$ function:

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha} B^{1-\alpha} K(t)^{\phi(1-\alpha)}.$$

so that the dynamics of $K(t)$ is given by

$$\dot{K}(t) = sB^{1-\alpha} K(t)^{\alpha+\phi(1-\alpha)} L(t)^{1-\alpha}$$

The returns to scale depend on how $\alpha + \phi(1-\alpha)$ compares with 1 or on how ϕ compares with 1. A steady state exists if either $\phi < 1$ or $\phi = 1$ and $n = 0$. In all other cases, growth is explosive.

In the case with $\phi = 1$ and $n = 0$:

$$Y(t) = (BL)^{1-\alpha} K(t)$$

and

$$\dot{K}(t) = s(BL)^{1-\alpha} K(t)$$

Capital grows at the constant rate $g_K = s(BL)^{1-\alpha}$. Output grows at the same rate. The long-term growth rate of the economy depends on the savings rate. This is because capital makes two-fold contribution to economic growth: savings raise output through the higher stock of capital and through the higher stock of knowledge.

Technological Progress and Population Growth over Very Long Run

Kremer (1993): model of endogenous accumulation of knowledge and population growth since 1,000,000 BC.

Most endogenous-growth models predict that technological progress increases in population size. If knowledge is *non-rival*, technological advances can be shared costlessly and a larger population size translates into more technological advances. A stylized fact of history, on the other hand, is that output growth typically translates into population growth rather than growth of output per person.Kremer's model combines these two features. The result is that the rate of population growth should have been rising over time: technological progress leads to higher growth which translates into population growth which in turn accelerates technological progress.

Production function:

$$Y(t) = T^{\alpha} [A(t) L(t)]^{1-\alpha}$$

where T is land (fixed), and for simplicity there is no capital. Accumulation of knowledge is proportional to population:

$$\dot{A}(t) = BL(t) A(t)^{\theta}.$$

Population growth is limited by the stock of land (Malthusian assumption):

$$\frac{Y(t)}{L(t)} = \bar{y}$$

where $\bar{y}$ is the subsistence level of output per person.

Because of the Malthusian assumption, population at any point in time is equal to the number that can be supported by the fixed amount of land, given the technology available at the time:

$$\begin{aligned} \frac{T^{\alpha} [A(t) L(t)]^{1-\alpha}}{L(t)} &= \bar{y} \\ T^{\alpha} A(t)^{1-\alpha} L(t)^{-\alpha} &= \bar{y} \end{aligned}$$

or

$$L(t) = \left(\frac{1}{\bar{y}}\right)^{\frac{1}{\alpha}} A(t)^{\frac{1-\alpha}{\alpha}} T.$$

Hence, population at any point in time is proportional to the amount of land, increasing in technology and decreasing in the subsistence level.

Growth rate of population is:

$$\frac{\dot{L}(t)}{L(t)} = \frac{1-\alpha}{\alpha} \frac{\dot{A}(t)}{A(t)}$$

(note: because $\bar{y}$ and T are both constant, $L(t)$ grows at the same rate as $A(t)^{\frac{1-\alpha}{\alpha}}$).

Special Case: $\theta = 1$

$$\frac{\dot{A}(t)}{A(t)} = BL(t)$$

$$\frac{\dot{L}(t)}{L(t)} = \frac{1-\alpha}{\alpha} BL(t)$$

so that population growth is proportional to the level of population.

General Case:

$$\begin{aligned} \dot{A}(t) &= BL(t)\, A(t)^{\theta} \\ \frac{\dot{A}(t)}{A(t)} &= BL(t)\, A(t)^{\theta-1} \end{aligned}$$

and

$$\begin{aligned} T^{\alpha} A(t)^{1-\alpha} L(t)^{-\alpha} &= \bar{y} \\ A(t)^{1-\alpha} &= \bar{y} T^{-\alpha} L(t)^{\alpha} \\ A(t) &= \bar{y}^{\frac{1}{1-\alpha}} T^{\frac{-\alpha}{1-\alpha}} L(t)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

This can be substituted into the expression for $\frac{\dot{A}(t)}{A(t)}$

$$\begin{aligned} \frac{\dot{A}(t)}{A(t)} &= BL(t) \left(\bar{y}^{\frac{1}{1-\alpha}} T^{\frac{-\alpha}{1-\alpha}} L(t)^{\frac{\alpha}{1-\alpha}} \right)^{\theta-1} \\ &= B \left(\bar{y}^{\frac{1}{1-\alpha}} T^{\frac{-\alpha}{1-\alpha}} \right)^{\theta-1} L(t)^{1-\frac{\alpha(1-\theta)}{1-\alpha}}. \end{aligned}$$

Therefore,

$$\frac{\dot{L}(t)}{L(t)} = \frac{1-\alpha}{\alpha} \tilde{B} L(t)^{\psi}$$

where $\tilde{B} = B\left(\bar{y}^{\frac{1}{1-\alpha}} T^{\frac{-\alpha}{1-\alpha}}\right)^{\theta-1}$ is a constant and $\psi = 1 - \frac{\alpha(1-\theta)}{1-\alpha}$. Hence, population growth increases in population also in the general case, unless α is very large or θ is much smaller than 1 or both.

Kremer tests the model using estimates of worldwide population going back to 1,000,000 BC and finds that population growth is indeed closely correlated with the stock of population.

As an alternative test, Kremer uses the fact that the various continents were isolated from each other since the end of the last Ice Age until approximately 1500. The model suggests that at the time of the separation, all continents had approximately the same level of technology and the same population density. Since separation, technological growth should have been faster in regions with larger population, in Europe-Asia-Africa, than in the Americas, Australia, and Tasmania. Therefore, population density at 1500 should be proportional to the land area (and thus the pre-separation stock of population), which is confirmed by the available estimates: 4.9 people per sq.km in Europe-Asia-Africa, 0.4 in the Americas, and 0.03 in both Australia and Tasmania.

The model also sheds light on why, by 1500, Europe-Asia-Africa was much more developed than the Americas, Australia, or Tasmania (although, obviously, technological progress was not uniform throughout Europe-Asia-Africa).

Jared Diamond (1999, Guns, Germs, and Steel) argues that rain forests in tropical Africa and Central/South America presented a largely impenetrable barrier to the spread of technology (as well as of agricultural crops and livestock). This would further limit the effective size of North/South America and Sub-Saharan Africa and contribute to their low population and underdevelopment relative to Europe-Asia-North-Africa